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Natural convection of water–fine particle suspension in a rectangular vessel heated and cooled from opposing vertical walls (classification of the natural convection in the case of suspension with a narrow-size distribution)

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Abstract

The water–fine SiO_2 particle suspension with a narrow-size distribution (mean diameter of 2.97 μm, standard deviation of 0.03 μm) was heated in a rectangular vessel from a vertical wall and cooled from the opposing vertical wall. Temperature distribution and local particle concentration of the suspension were measured under various temperature differences between the opposing vertical walls. It was found that behaviors of natural convection in the suspension were classified into five patterns. The critical wall temperature difference that would give rise to change of the natural convection pattern decreased as the initial particle concentration decreased.

Keywords: Natural convection; Suspension; Narrow-size distribution; Sedimentation; Stratification; Rectangular vessel

1. Introduction

A fundamental understanding of natural convection of emulsion or suspension is required for efficient design of various processes in chemical industry or foods industry and also in thermal energy utilization, for example, heat pump system and thermal energy storage system. When muddy water is used as a heat source for a heat pump system, the heat transfer should be treated as that of suspension. In a melting process of clathrate which formed from water and hydrofluorocarbon (HFC) mixture as a cold energy storage material, emulsion consisting of water and the HFC droplets dispersed in it formed and it was shown that the natural convection of the emulsion is different from that of a normal single phase liquid [1]. The natural convection occurring in suspension or emulsion shows complicated phenomena because of the interference of two types of density distributions. One is the unstable density distribution of liquid due to temperature difference and the other is the stable distribution of particle or droplet concentration due to the sedimentation (if the particle or droplet density is greater than the liquid one). A few studies were reported on the phenomena related to the natural convection of suspension. Okada and Suzuki [2] investigated a natural convection of a fine-particle distributed suspension in a rectangular vessel. They elucidated that the interference of the two density distributions helps to form the layers of convection cell, which is similar to the double diffusive convection in aqueous solution [3]. Chen et al. [4] investigated the process of vanishment of a formed layer and measured velocity distribution by means of visualization technique when a suspension consisting of water and glass beads with a few scores microns diameter was heated from bottom wall and cooled from two vertical walls. Okada

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Nomenclature

T	temperature (°C)
t	time (s)
Greek symbols	
ρ	particle concentration in suspension; density (kg/m^3)
Δ	difference of property
Subscripts	
c	cooling wall
h	heating wall
i	initial state

et al. [2,5,6] examined the effect of the size distribution on the mechanism of formation and vanishment of layer in suspension, using a suspension which consisted of water or aqueous solution and glass beads with a wide-size distribution ($\phi 1 \sim 30$ μm). In their reports, several items were measured, such as maintaining time of layer, variation of heat transfer coefficient on the heating wall due to the vanishment of layer, particle concentration in each layer, mean diameter and temperature distribution. Moreover, Okada et al. [7] investigated a natural convection of a suspension in a rectangular vessel when the suspension was heated from a vertical wall and cooled from the opposing vertical wall. In the report, the following results were found: (a) Multiple layers formed in a suspension that had a wide-particle size distribution. (b) The sedimentation velocity decreased as the initial particle concentration increased. (c) The lower layers had the larger particle concentration, the larger particle mean diameter and the larger sedimentation velocity. (d) The number of the formed layer was restrained in the case of a suspension with a narrow particle-size distribution. However, the influence of the temperature difference between the walls on the formation or vanishment of the layer was not investigated.

In the present study, we clarify the characteristics of the natural convection in a suspension, especially how the wall temperature difference affects the layer formation or vanishment. In order to neglect the effect of the particle-size distribution, the suspension that has a narrow particle-size distribution is used.

2. Experimental apparatus

Fig. 1 shows the experimental apparatus that is a rectangular vessel with 40 (w) × 160 (h) × 200 (b) mm^3. The suspension with a particle concentration was saturated in the vessel. Two vertical walls were made of copper plates. The left vertical wall was heated by hot water supplied from a high-temperature thermostatic bath and the right wall was cooled by cold water from a low-temperature thermostatic one. The other walls were made of transparent acrylic plates in order to observe the behavior of suspension by picture. Top and bottom of the rectangular vessel was insulated from heat transfer using polystyrene foam. Temperatures were measured by T-type thermocouples. The wall temperatures were measured on the surfaces contacting to the suspension. The temperature in suspension was measured with a thermocouple attached to a movable slender bar. The slender bar was fixed on a precise traversing device set above the test vessel. The thermocouple on the bar was moved in millimeter scale to measure the vertical temperature distribution of suspension along the centerline. In order to reduce the effect of flow induced by bar movement, the bar was moved slowly from point to point during 5 s. It took about 400 s for 80 temperature measuring points from first point (top) to the last one (bottom). The measuring errors of the temperatures were ±0.1°C approximately.

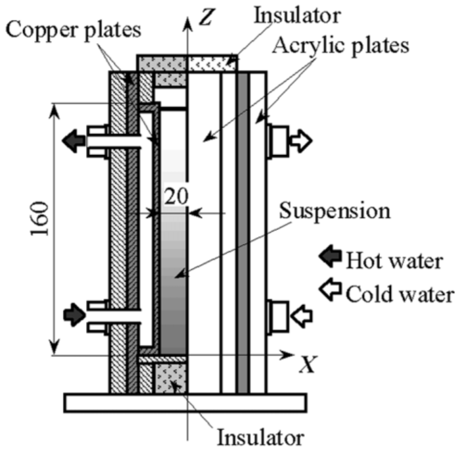

Fig. 1. Schematic of experimental apparatus.

3. Characteristics of suspension

Table 1 shows the properties of SiO_2 particles that were used to form a suspension (mixture of SiO_2-fine particles and pure water). The mean diameter of the particles is 2.97 μm with the standard deviation of 0.03 μm. Fig. 2 shows the picture of the powder with micro-scale and the particle-size distribution. From Table 1 and Fig. 2, it was found that the particles had a

Table 1
Narrow-size distribution SiO_2 powder

Temperature (°C)	Specific weight	Mean diameter (μm)	Standard deviation (μm)
20	2.15	2.97	0.03

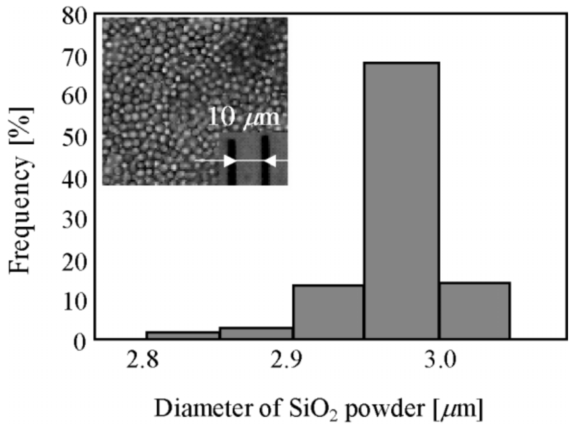

Fig. 2. The particles with narrow-size distribution (frequency, by courtesy of Ube Nitto Kasei).

narrow-size distribution, namely an almost uniform diameter.

4. Experimental method

The two kinds of suspension with different initial particle concentrations ρ_i of 5 and 10 kg/m^3 were used independently. The test vessel was kept initially at 10°C by supplying water with the constant temperature to the back of both the walls before it was filled with the well-mixed suspension at 10°C. After 300 s, the experiment started with supplying water at a predetermined high temperature to the back of the left wall. It was continued to supply water at 10°C to the back of the right wall. Although both the wall temperatures depended on the temperature of supplied hot water, they were constant during the experiment. Temperatures at three points which located on a diagonal line on each vertical wall were measured intermittently. It was found that the temperatures at the three points on each wall were distributed with the standard deviation of 0.25°C during experiments. The average temperatures of each wall, T_h and T_c, are shown in Figs. 3–5 and Figs 10–12. It is found that the T_h and T_c, were constant with time, respectively. The temperature and the particle concentration of suspension were measured. Let us define the interface as a zone with a few millimeters width between the formed layers. The position and the behavior of interface were investigated with photograph by (video) camera. The particle concentration was obtained by measuring weight, i.e., the powder remained by evaporating completely a sampled suspension was weighed by a precise electric balance with the 0.1 mg minimum scale.

5. Experimental results and discussion

5.1. The behavior of the layer formation and vanishment

In both the suspensions of $\rho_i = 5$ and 10 kg/m^3, three or two layers were observed in a certain time period after start of experiment and some complicated behaviors of layer vanishment were observed. The phenomena depended on the temperature difference between the vertical walls, ΔT. The phenomena

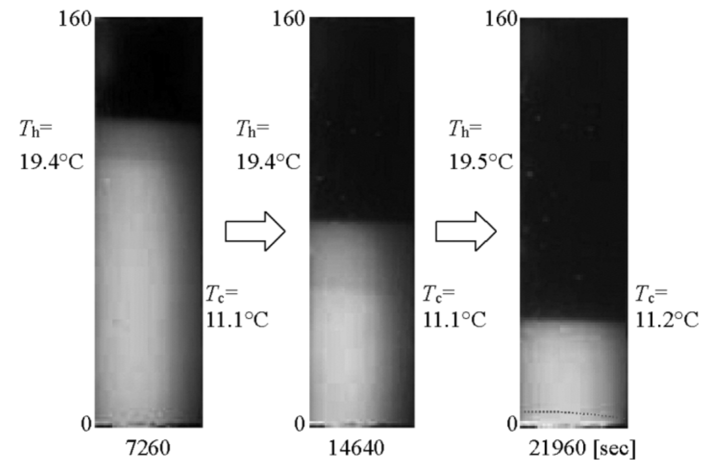

Fig. 3. Transient location of the interfaces in Case 1 (10 kg/m^3, $\Delta T = 8.3$°C).

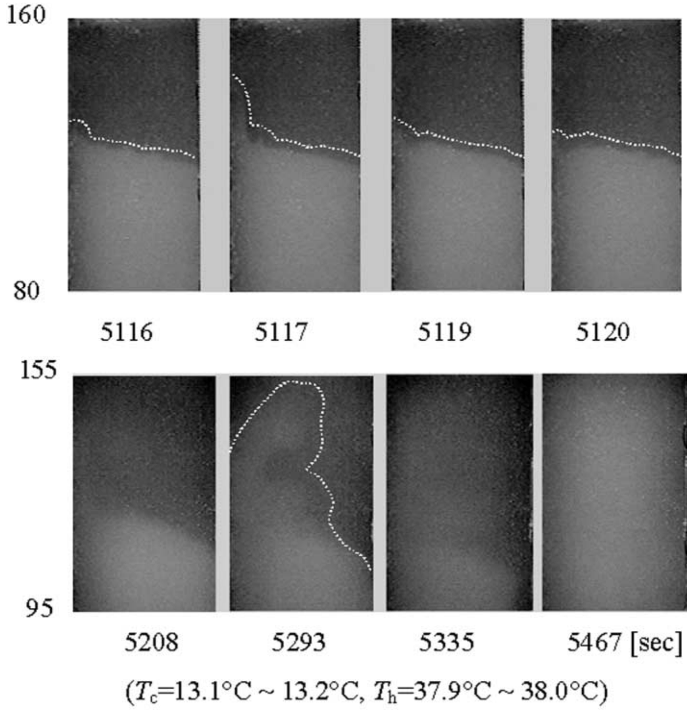

Fig. 4. Movement of interface in Case 3 (10 kg/m^3, $\Delta T = 24.8$°C).

of the formation and vanishment of the layers observed from the experiments were classified into the following five patterns. The patterns occurred in order for both the concentrations of 5 and 10 kg/m^3 as the ΔT increased;

Case 1: Three or four layers occurred. After that, the three (four) layers became two (three) layers and finally one layer as the lower layer sedimented orderly to bottom. Fig. 3 shows one of Case 1.

Case 2: Two layers occurred. The interface between the layers went down to the bottom with time.

Case 3: Two layers occurred. The interface between the layers undulated. The end of the interface near the cold wall (right wall) went down abruptly along the cold wall and the other end of the interface near the hot wall (left wall) went up abruptly. The suspension in the lower layer flowed near the hot wall into the upper layer and then both the layers mixed to be one layer. Case 3 is shown in Fig. 4.

Case 4: Two layers occurred. The interface undulated. Although the interface went down, in the middle of sedimentation, it rose up quickly to the top surface and finally vanished. Case 4 is shown in Fig. 5.

Case 5: Only one layer existed at all times.

For $\rho_i = 5$ and 10 kg/m^3, Table 2 shows the state of the interface and the number of the formed layer under

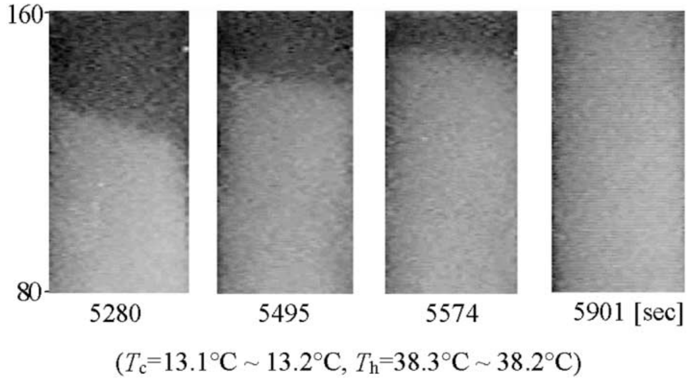

Fig. 5. Transient location of the interface in Case 4 (10 kg/m^3, $\Delta T = 25.2$°C).

Table 2
State of interface in suspension ($\rho_i = 5$ and 10 kg/m^3)

ΔT (°C)		State of interface	Pattern of sedimentaion	Number of formed layer
5 kg/m^3	10 kg/m^3			
4.2	8.3, 12.3	Sedimentation	Case 1	3, 4
8.4, 12.4	16.8, 24.0	Sedimentation	Case 2	
14.3, 15.1	24.8	Mixing up locally	Case 3	2
15.9	25.2, 25.5	Rapid ascending	Case 4	
17.1	27.8	No interface	Case 5	1

each condition of ΔT. In the present paper, we numbered the layers and the interfaces from top in turn.

When the temperature difference was $\Delta T = 8.3$°C and 12.3°C for the initial concentration $\rho_i = 10$ kg/m^3, three layers formed as shown in the Table 2 (Case 1). Three layers are distinguishable with the shades of color. When $\Delta T = 8.3$°C, the pictures of the formed layers are shown in Fig. 3. In Fig. 3 the second interface between the third layer (the brightest region) and the second layer (middle-dark region) went down to the bottom with time. The second interface (at time of 21,960 s) was indicated by a dotted line so as to make it clear. The time history of the positions of the interfaces is shown in Fig. 6. From Fig. 6, the sedimentation velocity of the second interface was faster than that of the first one. When $\Delta T = 16.8$°C and 24.0°C for $\rho_i = 10$ kg/m^3 in Table 2 (Case 2), two layers formed and the interface went down gradually and finally vanished. It was observed that the slope of the interface became steeper as the wall temperature difference ΔT became larger. The transient locations of the first interface in Cases 1 and 2 are shown in Fig. 7 for $\rho_i = 10$ kg/m^3 and in Fig. 8 for $\rho_i = 5$ kg/m^3. It is found from Figs. 7 and 8 that the interface went down to bottom and that the sedimentation velocity was constant for each temperature difference. When $\Delta T = 24.8$°C, the interface could not be kept stable because the convection due to water density change in the each layer became more active, and hence

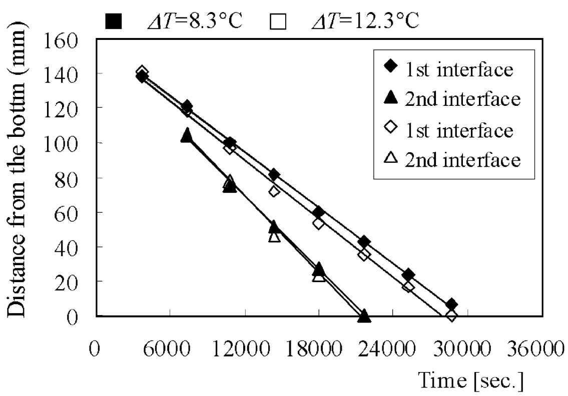

Fig. 6. Transient location of interfaces, Case 1 (10 kg/m^3).

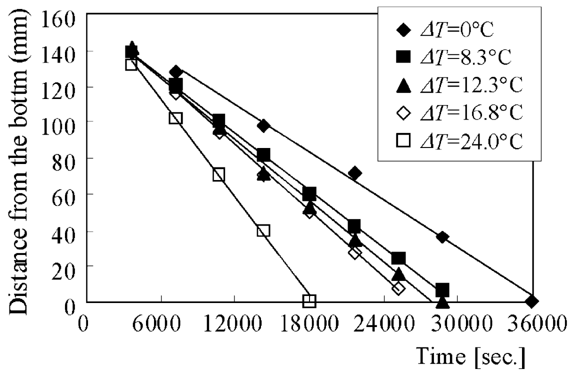

Fig. 7. Transient location of the first interface (10 kg/m^3).

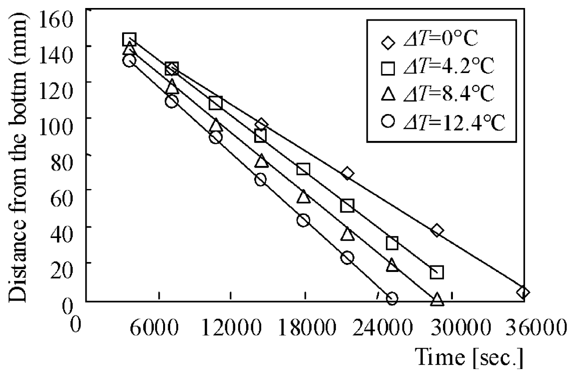

Fig. 8. Transient location of the first interface (5 kg/m^3).

the collapse of layers which was classified into Case 3 was found. Fig. 4 represents well the phenomena of Case 3. The pictures at every $t = 5116$ to 5120 s in Fig. 4 show undulations of the interface which appeared before mixing. The collapse of the layers happened at 5293 s as the interface was shown with the dotted line. Namely, the upward flow near the hot wall from the lower layer to the upper layer mixed both the layers due to the locally density difference near the wall. On the other hand, the result of $\Delta T = 25.2$°C is shown in Fig. 5. Although $\Delta T = 25.2$°C was almost same as $\Delta T = 24.8$°C, the

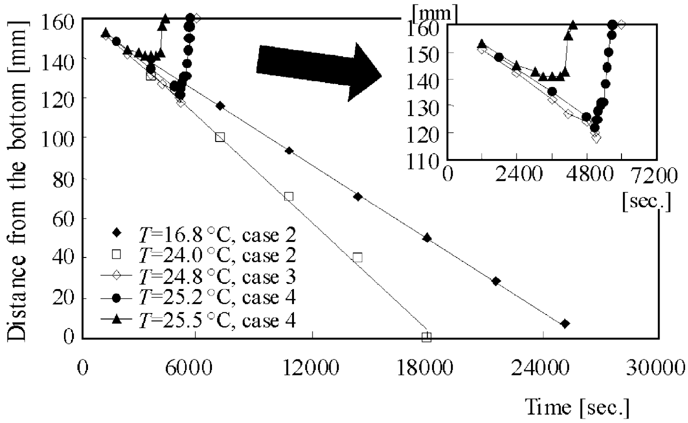

Fig. 9. Transient location of the first interface (Cases 2, 3 and 4).

interface in the case of $\Delta T = 25.2°C$ went down before turning upward in 5280 s and then continued to rise to the top for 600 s. This is Case 4. Fig. 9 shows the transient location of the first interface in Cases 2, 3 and 4. In Case 2 the first interfaces for $\Delta T = 16.8°C$ and 24.0°C sedimented to bottom with constant velocities. Meanwhile, in Case 4, the interface for $\Delta T = 25.2°C$ went down initially and turned upward at $t = 5400$ s in the middle of sedimentation. When ΔT increased to 25.5°C (Case 4), the turning time of the interface became shorter from 5400 to 3600 s. In Fig. 9 when $\Delta T = 24.8°C$ in Case 3, the sedimentation velocity and the maximum sedimentation location of the interface were, respectively, almost equal to those for $\Delta T = 25.2°C$ in Case 4 because both the ΔT's were almost equal to each other. The interface rising-up would be come from the uniform density distribution due to natural convection. When $\Delta T = 27.8°C$ corresponding to Case 5, the suspension was maintained to a single layer and the particle concentration was getting weak with time. The particle concentration will be mentioned later in detail. As the temperature difference became larger, the phenomena from Cases 1–5 occurred in order. The similar phenomena were observed for the experiments for $\rho_i = 5$ kg/m^3.

5.2. Temperature distribution in the vertical direction

In order to clarify the correlation between layer and temperature distribution, the temperature distribution of suspension along the vertical direction was measured with 2 mm intervals. Fig. 10 shows the temperature distribution of the suspension under $\rho_i = 10$ kg/m^3 and $\Delta T = 8.3°C$. The horizontal lines in Fig. 10 indicate the position of the interface determined from the pictures as shown in Fig. 3. The temperature in each layer increased with height and the temperatures across the interface decreased upward abruptly. The position of the interface predicted from the pictures indicates the upper bounds of the interface zone. The temperature distribution of Fig. 10 shows that a circulation flow must exist in each layer. The temperature distributions of Cases 2 and 3 are

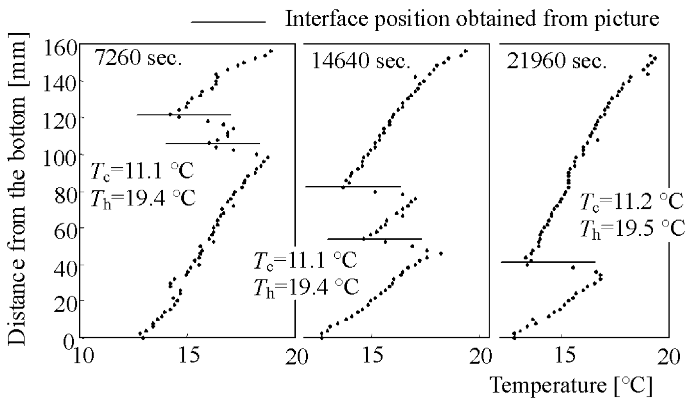

Fig. 10. Temperature distribution along the center of the vessel, Case 1 (10 kg/m^3, $\Delta T = 8.3°C$).

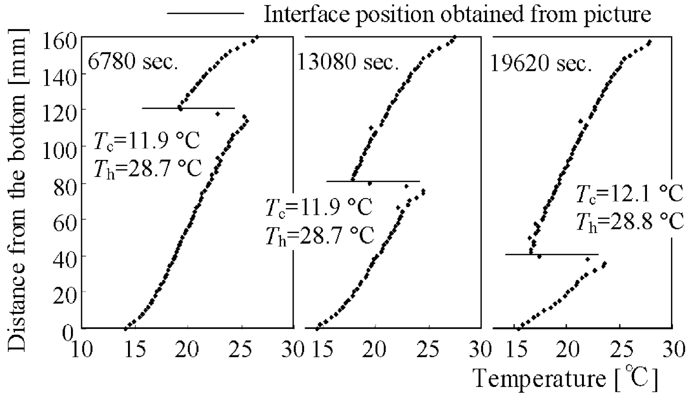

Fig. 11. Temperature distribution along the center of the vessel, Case 2 (10 kg/m^3, $\Delta T = 16.8$°C).

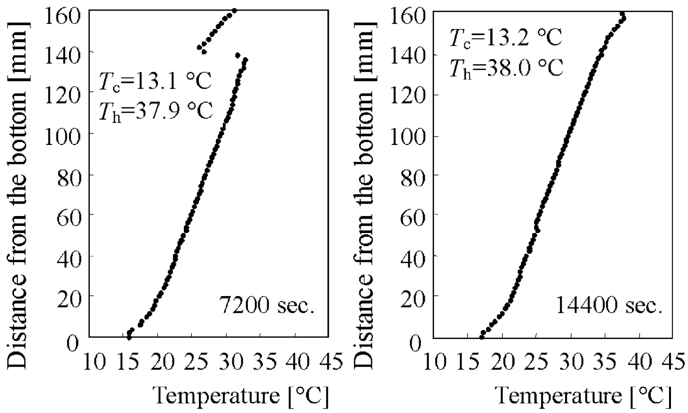

Fig. 12. Temperature distribution along the center of the vessel, Case 3 (10 kg/m^3, $\Delta T = 24.8$°C).

shown in Figs. 11 and 12, respectively. Since the strength of convection depends on the length of the heating or cooling surface, the maximum temperature difference in each layer became smaller as the heating or cooling wall area became smaller.

The temperature distribution in the suspension of $\rho_i = 5$ kg/m^3 had the similar tendency to the case of $\rho_i = 10$ kg/m^3.

5.3. Particle concentration in layer

Local distribution of particle concentration was measured in each layer. Fig. 13 shows the distributions of particle concentration in the suspensions with $\rho_i = 10$ kg/m^3 and $\Delta T = 8.3$°C. The horizontal (solid or dotted) lines indicate the position of the interface at each time. The step-like distribution of the particle concentration occurred as shown in Fig. 13. The time variation of the concentration in Fig. 13 was small for the first and third layers. And the concentration of the upper region in the second and third layer had a tendency to decrease with time. Moreover, the concentration of the second layer in Fig. 13 (Case 1) increased with time from 8580 to 16,500 s. Since the thickness of the second layer increased during this period, it is considered that the

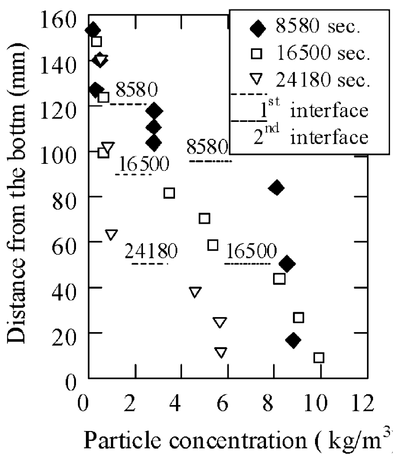

Fig. 13. Distribution of particle concentration on the center of the vessel (10 kg/m^3, $\Delta T = 8.3$°C, Case 1).

increase of the concentration in the second layer was caused by particle-transfer from the third layer to the second layer according to the sedimentation of the second interface. Figs. 14 and 15 show the particle concentration distributions in Cases 3 and 4, respectively, where $\rho_i = 10$ kg/m^3. In Figs. 14 and 15, two layers were still maintained at 3600 s. After the two layers changed to one layer, the distributions of the particle concentration were nearly constant over the suspensions. Although the pattern of the vanishment of layer was distinguished into Cases 3 and 4, the differences of the temperature distribution and of the concentration distribution between the two cases were not found. Fig. 16 shows the distribution of particle concentration in Case 5 with $\rho_i = 10$ kg/m^3 and $\Delta T = 27.8$°C. The particle concentration decreased gradually with time.

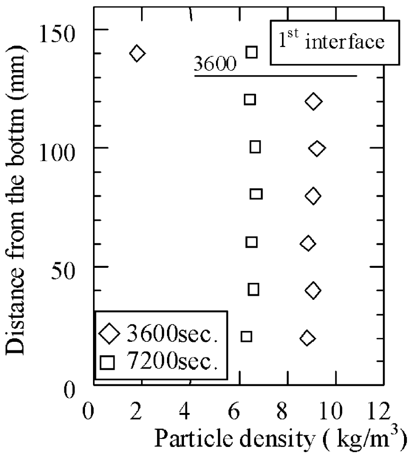

Fig. 14. Distribution of particle concentration on the center of the vessel (10 kg/m^3, $\Delta T = 24.8$°C, Case 3).

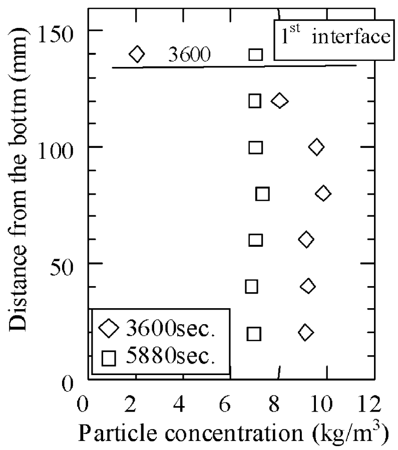

Fig. 15. Distribution of particle concentration on the center of the vessel (10 kg/m^3, $\Delta T = 25.2$°C, Case 4).

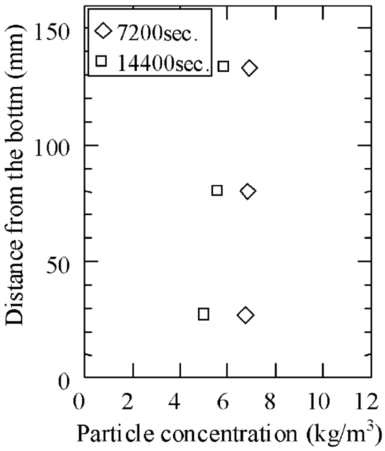

Fig. 16. Distribution of particle concentration on the center of the vessel (10 kg/m^3, $\Delta T = 27.8$°C, Case 5).

Under the large temperature difference a single circulation by convection was generated in the suspension without forming an interface, and particles sedimented gradually to bottom. Moreover, it was found from Figs. 13–16 that the particle concentration of the first layer increased as ΔT increased.

5.4. The natural convection phenomena of the suspension with the narrow-size distribution

It was suspected that the suspension with the narrow-size distribution particles would have only one interface during the sedimentation if the sedimentation velocities of particles should be equal to each other. Nevertheless, three layers formed in Case 1. This result suggests that the distribution of particle concentration increases downwards (in gravitational direction) at the initial state. Thereupon, the interface position and the particle concentration distribution were examined under the condition of $\Delta T = 0$°C ($T_c = T_w = 8.4$°C), i.e. without the effect of the natural convection. In this case, two layers are found. As shown in Figs. 7 and 8, when $\Delta T = 0$°C, the interfaces sedimented with constant velocities to the bottom for both the suspensions of $\rho_i = 5$ and 10 kg/m^3. The sedimentation velocities were nearly equal to each other. Fig. 17 shows the distribution of the particle concentration of the second layer where $\Delta T = 0$°C and $\rho_i = 10$ kg/m^3. The vertical lines in Fig. 17 indicate the measured interface at each time. It is found that the particle concentration decreased with time at each measuring point and had a negative gradient in the upward direction even if the well-dispersed suspension was left as it was. It is considered that the distribution would be caused from so-called hydrodynamic diffusion [8], i.e., if sparse particle groups are surrounded with upward stream which is induced by the

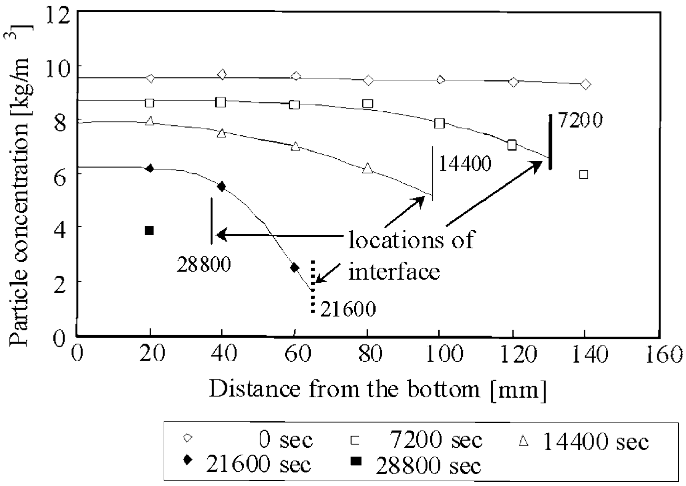

Fig. 17. Concentration distribution along the center of the vessel ($\Delta T = 0°C$).

sedimentation of dense particle groups, the sedimentation velocity of the sparse particle groups decreases and finally the concentration distribution forms. The particle concentration and the water density depending on temperature determine density of suspension. Since the suspension heated from the hot wall has less density of water, it flows upward. If the upper suspension has less particle concentration, the suspension will not continue flowing upward but flows horizontally to the cold wall. This is the reason why the layers appear. The number of layer depends on the particle concentration gradient and the temperature difference between the walls. The number will increase with increase of the particle concentration gradient or with decrease of the temperature difference. For example, there are the concentration gradients in the second layer in Fig. 17. This is the reason why three layers formed in Case 1. If the temperature difference is smaller than that of Case 1, it is possible to form more than three layers. In an experiment, when $\Delta T = 4.2°C$ and $\rho_i = 10$ kg/m^3, four layers were observed. The physical behavior in the four layers was similar to that in the three layers. Hence, the four layers can be included in Case 1. Whereas, in Case 2 (where ΔT was larger than that of the case 1), the two layers were formed from the balance between the unstable density stratification due to temperature distribution and the stable stratification due to particle concentration distribution. Although the interface reached the bottom in Cases 1 and 2, it vanished before reaching the bottom in Cases 3 and 4. The density difference of suspension between layers is considered to play an important role in classification of the natural convection. The density of suspension in each layer was obtained from the measurements of particle

Table 3
Density difference between the layers when $\rho_i = 10$ kg/m^3

Layer no. ΔT (°C)	1st layer (kg/m^3)	2nd layer (kg/m^3)	3rd layer (kg/m^3)	$\Delta\rho$ (kg/m^3)	Pattern of sedimentation
8.3	999.6	1001.6		2.0*	Case 1
	(7380 s)[a]	1001.8	1007.0	5.2**	
16.8	999.1	1006.2	–	7.1	Case 2
	(6780 s)[a]				
24.0	999.0	1004.7	–	5.7	Case 2
	(4980 s)[a]				
24.8	999.6	1004.9	–	5.3	Case 3
	(3600 s)[a]				
25.5	1000.2	1004.6	–	4.4	Case 4
	(1800 s)[a]				
27.8		–	–	–	Case 5

[a] Measured time.
* (= 1001.6–999.6).
** (= 1007.0–1001.8) (kg/m^3).

concentration and temperature in each layer. Table 3 shows the density in each layer. The values written below the densities of first layers indicate the time when each density was measured. From the table it is found that as the wall temperature difference became larger, the concentration difference between the upper and lower layer became smaller; namely, the convection velocity and the particle sedimentation rate became larger and then the particle concentration of the lowest layer became smaller. At the same time, the particle concentration in the first layer increased because particles were supplied from the lower layer with high concentration. Hence, the density difference between the upper and lower layers became smaller. The decrease of the density difference caused the collapse of the stable interface in the middle of sedimentation. After all, the two layers turned into one layer. In advance, in Case 5 where the temperature difference was the largest among the present experimental conditions, the consistent single convection might occur due to the influence of the large convection velocity.

The phenomena stated above were the same as those for $\rho_i = 5\ \mathrm{kg/m^3}$. As the initial concentration became smaller, the influence of the wall temperature difference became larger and, therefore, the temperature difference that changed the pattern of the layer occurrence or vanishment became smaller.

6. Conclusion

Experiments of natural convection were carried out for a suspension with a narrow-size distribution suspension in a rectangular vessel with heated and cooled opposing walls, and the following conclusions were obtained:

1. The distribution of particle concentration formed in suspension though the suspension had a narrow-size distribution.
2. In the natural convection of suspension, the formation and vanishment of layers occurred and they were classified into five patterns according to the increase of the wall temperature difference, as follows; (a) Three or four layers appeared. All the interfaces went down to the bottom. (b) Two layers appeared. The interface went down to the bottom. (c) Unstable two layers appeared. The upper layer mixed with the lower one in the middle of sedimentation. (d) Unstable two layers appeared. The interface went down and turned upward in the middle of sedimentation and then it rose up rapidly to the top. (e) Only one layer appeared. The layer remained all the time.
3. As the initial particle concentration became smaller, the critical wall temperature difference that distinguished the pattern of the formation or vanishment of layers became smaller.

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